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# ULTRARELATIVISTIC GAS SCATTERING IN THE GRAVITATIONAL FIELD

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## SUMMARY

This paper considers the scattering of gas in the general theory of relativity, when the gas velocity is quite near the speed of light. Interest in this problem is aroused in connection with the observations of supernovae and explosions of nuclei of galaxies. A relatively simple method is proposed for the clarification of entire motion pattern.

* * *

The problem described offers also an independent interest as an example of statement of the Cauchy problem for Einstein equations.

The scattering of ultrarelativistic gas in the special theory of relativity was considered in the works by Landau [1], Khalatnikov [2], Stanyukovich [3] and others in connection with the problem of multiple particle formation at collisions of superfast nucleons. For the application of methods developed by the indicated authors it is necessary to write the equations of motion of the gas in the general theory of relativity, similar in form to equations of the special theory of relativity. The latter is attained by the selection of the centrally-symmetrical interval

$$ds^2 = e^{\nu} d\tau^2 - r^2 d\Omega^2 - e^{\lambda} dr^2 \tag{1}$$

and by writing the equations of motion and continuity in the form [3, 4]:

$$\begin{gathered}\frac{1}{\theta^2}\left(A\frac{\partial a}{\partial \tau} + a\frac{\partial a}{\partial r}\right) + \left(\frac{\partial \ln w}{\partial r} + aA\frac{\partial \ln w}{\partial \tau}\right) + \frac{1}{2}\left(aA\frac{\partial \lambda}{\partial \tau} + \frac{\partial \nu}{\partial r}\right) = 0, \\ \frac{1}{\theta^2}\left(Aa\frac{\partial a}{\partial \tau} + \frac{\partial a}{\partial r}\right) - \left(A\frac{\partial \ln v}{\partial \tau} + a\frac{\partial \ln v}{\partial r}\right) + \frac{2a}{r} + \frac{1}{2}\left(A\frac{\partial \lambda}{\partial r} + a\frac{\partial \nu}{\partial r}\right) = 0.\end{gathered} \tag{2}$$

* UL'TRARELYATIVISTKIY RAZLET GAZA V POLE GRAVITATSII

Here $v$ is the specific volume, $w = v(p+\varepsilon)$ is the heat content per unit of mass, a is the root-mean-square velocity measured by its proper time; $\theta^2 = 1 - a^2$; $A = \exp[(\lambda - \nu)/2]$; we assume the flow to be isentropic, that is, $\sigma = \text{const}$. We postulate $c = 1$ in the expressions (1) and (2); at the same time, a is dimensionless and the time $\tau$ has the dimensionality of the length.

The system (2) is complemented by the Einstein field equations

$$e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = \varkappa\frac{(p+\varepsilon a^2)}{\theta^2}; \quad e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = \varkappa\frac{(\varepsilon + a^2 p)}{\theta^2} \tag{3}$$

or

$$Ae^{-\lambda}\frac{\partial\lambda}{\partial\tau} = -\frac{\varkappa(p+\varepsilon)ar}{\theta^2}.$$

For the ultrarelativistic velocity, when

$$1 - a = 2\Delta, \quad \Delta \ll 1,$$

and the equation of state $p = (k-1)\varepsilon$ (subsequently it is postulated $k = 4/3$ for formula contraction, although other values of k do not complicate the solution), we shall have a significantly simpler initial system. The initial distribution of pressure and velocity (the Cauchy problem) is also appropriate to be chosen simplified, as this is done, for example, at automodel investigation of a common gas [5].

We shall assume that at the initial moment of time ($\tau = 0$) there is a gas sphere of radius R and constant pressure $p_0$, in which a constant velocity $\Delta_0$, directed from the center, is given. Since the free gravitational field is absent at centrally-symmetrical motion, the given distribution of matter determines also the metric at the initial moment of time

$$\lambda = -\ln\left(1 + \frac{\varkappa}{r}\int_0^r T_0^0 r^2 dr\right), \quad \lambda + \nu = \varkappa\int_0^r (T_1^1 - T_0^0)\, re^{\lambda} + C. \tag{5}$$

Here $\varkappa$ is the Einstein constant; C is the integration constant determined from the condition that at the initial moment of time $r = R$, $\lambda + \nu = 0$., as this follows from the Schwarzshield metric. At the same time, it follows from (5)

$$e^{-\lambda} = 1 - \frac{\varkappa p_0 r^2}{3\Delta_0}, \quad e^{\nu} = \left(1 - \frac{\varkappa p_0 R^2}{3\Delta_0}\right)^3 e^{2\lambda}, \tag{6}$$

$$e^{-\lambda} = 1 - \frac{\varkappa p_0 R^2}{3\Delta r} \text{ at } r < R; \qquad e^{\nu} = e^{-\lambda} \text{ ar } r > R.$$

Upon a series of transformations the ultrarelativistic approximation allows to obtain from the systems (2) and (3) the equations

$$A\frac{\partial p}{\partial \tau} + \frac{\partial p}{\partial r} = -\frac{4p}{r}, \quad A\frac{\partial \lambda}{\partial \tau} + \frac{\partial \lambda}{\partial r} = -\left(\frac{e^{\lambda} - 1}{r} + \varkappa r p e^{\lambda}\right),$$
$$A\frac{\partial \psi}{\partial \tau} + \frac{\partial \psi}{\partial r} = \frac{1}{2p}\left(A\frac{\partial p}{\partial \tau} + \frac{\partial p}{\partial r}\right), \quad \psi = \ln \Delta + \lambda. \tag{7}$$

The system obtained is practical in that the unknown $\nu$ enters in only $A = \exp[(\lambda - \nu)/2]$ and the elimination of $\nu$ allows to find the solution of part of the equations, without resorting to those not written in (7).

The elimination of A is obtained by transition to new independent variables p and r. At the same time, the first equation of (7) has the form:

$$A - \frac{\partial \tau}{\partial r} = -\frac{4p}{r}\frac{\partial \tau}{\partial p}. \tag{8}$$

With the help of (8) we obtain for the two other equations (7)

$$\frac{\partial \psi}{\partial r} - \frac{4p}{r}\frac{\partial \psi}{\partial p} = -\frac{2}{r}, \quad e^{-\lambda}\left(\frac{\partial \lambda}{\partial r} - \frac{4p}{r}\frac{\partial \lambda}{\partial p}\right) = -\left(\frac{1 - e^{-\lambda}}{r} + \varkappa r p\right). \tag{9}$$

For the first integrals of the equations (9) we have the expressions

$$C_{1\lambda} = pr^4, \quad C_{2\lambda} = r(1 - e^{-\lambda}) - \varkappa p r^3, \quad C_{1\psi} = pr^4, \quad C_{2\psi} = \Delta e^{\lambda} p^{-1/2}. \tag{10}$$

Hence it may be seen directly that at scattering of the ultrarelativistic gas in the gravitational field the pressure p drops by $r^{-4}$, that is, just like in the special theory of relativity. The solutions for $\lambda$ and $\Delta$ satisfying the above-given initial conditions, are found with the aid of (10) by the standard method*

$$e^{-\lambda} = 1 - b\left(\frac{p}{p_0}\right)^{3/4} r^2, \quad \Delta = \Delta_0\left[\frac{1 - b(p/p_0)^{3/4} r^2}{1 - b(p/p_0)^{1/2} r^2}\right]\left(\frac{p}{p_0}\right)^{1/2}, \quad b = \frac{\varkappa p_0}{3\Delta_0}. \tag{11}$$

* When obtaining (11), $\varkappa p r^3$, which is small by comparison with the terms containing b, was dropped.

Therefore, at gas scattering the metric approaches the Euclidean as $(p/p_0)^{3/4}$. In case of absence of gravitational forces the velocity $\Delta$ would rise as $(p/p_0)^{1/2}$, as it follows from (11). Hence the quantity in brackets determines the character of velocity deceleration in the gravitational field.

For the determination of $\nu$, we have from the first two equations of the system (3)

$$2A\frac{\partial\lambda}{\partial p} = -\frac{\partial\tau}{\partial p}\frac{\partial\varphi}{\partial r} + \frac{\partial\tau}{\partial r}\frac{\partial\varphi}{\partial p}, \quad \varphi = \lambda + \nu. \tag{12}$$

Eliminating $\partial\tau/\partial r$ and $\partial\tau/\partial p$, from (12) with the aid of the third equation of (3) and of (8), we shall obtain

$$2\frac{\partial\lambda}{\partial p} - \frac{\partial\varphi}{\partial p} = e^{-\lambda}\frac{\partial\lambda}{\partial p}\frac{\Delta}{\varkappa pr}\left(\frac{\partial\varphi}{\partial r} - \frac{4p}{r}\frac{\partial\varphi}{\partial p}\right). \tag{13}$$

In the latter equation $\lambda$ and $\Delta$ have already been found. Determining $\nu = \varphi - \lambda$ from (13) we shall find with the aid of (8) $\tau = \tau(r, p)$. Such is the solution scheme for the problem as first proposed in [6].

Because of its cumbersomeness the solution of (13) in an analytical form is not attainable, which compels us to find $\nu$ and $\tau$ by numerical integration.

However, there exists a comparatively simple method for ascertaining the whole pattern of motion. Let us consider first of all the scattering of ultrarelativistic gas in the outer Schwarsshield field. Such a problem describes the scattering of the relativistic star's shell after emergence on the surface of the shock wave moving from the center. Compared to the mass of the remnant star M, the mass of the scattering gas is neglectingly small. That is why the gas gas flows in the outer Schwarsshild field created by the star

$$e^{-\lambda} = e^{\nu} = (1 - r_g/r), \quad r_g = 2kM/c^2. \tag{14}$$

If at $\tau = 0$ the gas pressure in a shell of radius R were $p_0$ and the velocity $\Delta_0$, we would have for the velocity (rate) of subsequent scattering at $r > R$ from the equation (10) with the help of (14) :

$$\Delta = \Delta_0\left[\frac{1 - r_g/r}{1 - r_g/R}\right]\left(\frac{p}{p_0}\right)^{1/2}. \tag{15}$$

and it follows from (8) for the time $\tau$ at $\Lambda = (1 - r_g/r)^{-1}$

$$\tau = r\left[1-\left(\frac{p}{p_0}\right)^{1/4}\right] + r_g \ln\left[\frac{r-r_g}{r\,(p/p_0)^{1/4}-r_g}\right]. \tag{16}$$

When $r_g$ approaches zero, it follows from (15) and (16) the well known result for gas scattering in the special theory of relativity

$$\Delta = \Delta_0\left(\frac{p}{p_0}\right)^{1/2}, \quad \left(\frac{p}{p_0}\right) = \left(\frac{r-\tau}{r}\right)^4.$$

Let us now turn back to the initial problem of gas sphere scattering in proper gravitation field. Because the matter distribution inside the sphere is manifest on the metric outside it only integrally, we shall estimate, in accord with (10) that at $r > R, (p/p_0) \sim (R/r)^4$; at the same time, for the velocity (11) we shall have

$$\Delta \cong \Delta_0\left[\frac{1-bR^3/r}{1-bR^3/R}\right]\left(\frac{p}{p_0}\right)^{1/2}, \tag{17}$$

which coincides with (15). Hence it follows that the ultrarelavistic scattering of an initially uniform gas sphere of radius R at r > R proceeds just as effectively as at gas flow in the outer Schwarsshild field with a gravitational radius $r_g = r_g^* = \varkappa p_0 R^3/3\Delta_0$; at the same time $e^{-\lambda} = (1 - r_g^*/r)$.

**** THE END ****

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